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The Law and Large Numbers

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THE LAW AND LARGE NUMBERS

POLITICAL NUMERACY: MATHEMATICAL PERSPECTIVES ON OUR CHAOTIC CONSTITUTION. By Michael I. Meyerson¹. W. W. Norton and Company. 2002. Pp. 9, 287.

*Paul H. Edelman*²

Can mathematics be used to inform legal analysis? This is not a ridiculous question. Law has certain superficial resemblances to mathematics. One might view the Constitution and various statutes as providing “axioms” for a deductive legal system. From these axioms judges deduce “theorems” consisting of interpretation of these axioms in certain situations. Often these theorems are built on previously “proven” theorems, i.e. earlier decisions of the court. Of course some of the axioms might change, and occasionally a theorem that was once true becomes false; the former is a common feature of mathematics, the latter, though theoretically not possible in mathematics (since a theorem is by definition true) has been known to happen in mathematical practice as well.³

So maybe mathematics can help law scholars. That is certainly what Michael Meyerson believes. His new book is “premised on the belief that there are many legal ideas that can be explained or clarified by mathematics.” (p. 47) He presents an extended set of examples to illustrate how mathematics and mathematical thinking can be useful in understanding legal is-

1. Piper and Marbury Faculty Fellow and Professor of Law, University of Baltimore School of Law.

2. Professor of Mathematics and Law, Vanderbilt University. I have benefited from the comments of Mark Brandon, John Goldberg, Chris Guthrie, Robert Rasmusen, and Suzanna Sherry.

3. For example, the four color theorem was originally thought to be proven in 1879 by Kempe. The flaw in his proof wasn't recognized until 1890 by Heawood. See N. L. Biggs, E. K. Lloyd and R. J. Wilson, *Graph Theory 1736-1936* at 90 (Clarendon Press, 1976). A correct proof was finally announced by Appel and Haaken in 1976. See Kenneth. Appel and Wolfgang Haken, *Every Planar Map is Four Colorable, I & II*, 21 Ill. J. Math. 429 (1977).

sues. Some of his examples are persuasive indeed. Others are less compelling. In this review I will describe some of his examples and assess how much the mathematics really adds to the legal analysis.

WHAT CONSTITUTES MATHEMATICAL THINKING?

Before exploring Meyerson's examples it is worthwhile to consider what, exactly, it means to think mathematically about the law. There are a number of different things one might mean by this, although Meyerson treats them all the same. The persuasiveness of his applications of mathematics to legal analysis depends on which of these interpretations is being employed.

One can break down mathematical thinking in the law into three types: general logic, technical mathematics, and mathematical metaphor. By general logic I mean the use of standard deductive reasoning that might be taught in a formal logic class. For example, such claims as "If A implies B and B implies C then A implies C" or "If P implies Q then Q being false implies that P is false" fall into this category. The syllogisms that Meyerson discusses (p. 25), such as

1. All men are mortal.
2. Socrates is a man.
3. Therefore, Socrates is mortal.

are also examples of what I call general logic.

The second class of mathematical thinking is what I will refer to as technical mathematics. Here theorems from mathematics are employed to provide a solution to a legal question. Technical mathematics is illustrated by Meyerson's discussion of the mathematical methods employed in the apportioning of the House of Representatives (p. 82). What distinguishes this class of thinking from general logic is the level of mathematical sophistication that is being employed.

The final class of mathematical thinking is mathematical metaphor. In this style of thinking the ideas of mathematics are used as a source of inspiration but not put to any technical use. So when Meyerson discusses "constitutional topology" (p. 134) he does not intend to give a formal definition of the Constitution as a topological space, but rather to use the ideas of topology to give some informal description of certain properties of the Constitution, to wit:

Imagine the federal government as a sphere containing all of the powers granted by the Constitution. Within that sphere, however, there exists a hole, consisting of the powers that are, in the words of the Tenth Amendment, "reserved to the States." Over time, the size of the hole has grown and shrunk relative to the size of the sphere, but the hole must remain if the Constitution's topological structure is to remain intact (p. 138).

Clearly Meyerson does not believe that the Constitution is a sphere in any meaningful sense, but he finds the analogy of the sphere and the use of the language of topology illuminating.⁴

The remainder of this review will discuss Meyerson's uses of each of these types of mathematical thinking. Meyerson's use of general logic is impeccable, but I will question if its use is distinctly mathematical. His use of technical mathematics is the most interesting and persuasive part of the book. In addition to describing some of his applications, I will suggest some further applications of technical mathematics. Finally, I will cast doubt upon Meyerson's use of mathematical metaphor in understanding legal issues.

GENERAL LOGIC

Meyerson begins his foray into mathematical applications by considering the axiomatic nature of much of legal argumentation. He starts by analyzing the logical structure of the Declaration of Independence (p. 28) and continues through a dissection of the logical foundations of *Korematsu v. United States*.⁵ Along the way he finds an opportunity to discuss Euler's proof of the infinitude of prime numbers and Lindemann's proof that it is not possible to square a circle.⁶

4. Sitting astride technical mathematics and mathematical metaphor is mathematical modeling. For example, Meyerson analyzes Marshall's dilemma in *Marbury v. Madison* as a two-person game between Marshall and Jefferson. (p. 112) Now clearly Marshall and Jefferson were not literally playing such a game, but if the analogy between their situation and a two-person game is convincing, then one can employ a technical mathematical analysis of the game to shed light on the real-world events. For lack of space I will not consider these chapters in this review, except to remark that in terms of quality of insight they seem to behave more like mathematical metaphor than technical mathematics.

5. 323 U.S. 214 (1944).

6. Meyerson makes a small mathematical mistake in this section. He confuses constructible numbers, those numbers that can be constructed using a compass and a straight edge, with algebraic numbers, those that are the roots of polynomials with integer coefficients. All constructible numbers are algebraic, but the converse is not true. Given the context, this is a minor error.

The early part of the chapter demonstrates the influence that formal deductive proof had at the time of the Founders and how the structure of a formal proof was consciously mimicked by them in the Declaration of Independence. He goes on to show how formal deductive reasoning is evident in many Supreme Court opinions. I found this historical section particularly interesting.

Meyerson wants to make some stronger points as well. His main point is that all formal logical arguments start from the assumption of certain axioms. If these axioms are false, then the validity of the subsequent deductions is irrelevant to the truth of the final conclusion. By subjecting legal arguments to a close logical reading one can uncover the axioms underlying the argument and thus better understand the nature of the argument. He applies this technique to notorious Supreme Court decisions such as *Dred Scott*⁷ and *Korematsu*.

Meyerson's reminder that assumptions, both stated and unstated, are critical in deductive logic is an important one. It is often the first place to look in analyzing an argument. At the foundation of mathematical thinking is the ability to understand the difference between hypotheses and theorems, and so this is a fine beginning to Meyerson's task of demonstrating the importance of mathematical thinking to legal analysis.

It would have been helpful, though, if Meyerson had been more careful about two things. First, just because an argument is logically flawed, either because an assumption is false or because of some other logical error, it does not follow that the conclusion is false. Many scholars have critiqued the logic of *Roe v. Wade*⁸ but still support the result.⁹ So Meyerson's assertion that "Obviously, if the logical argument is not well-formulated, . . . the result is laughable," (p. 25) is simply not correct. It is one thing to say that the conclusion is not proved, quite another to claim that the conclusion is itself false.

The second point about which Meyerson could have been more careful is his use of the term axiom. Early in his discussion of formal logic Meyerson defines an axiom as "a statement used in the premises of arguments and assumed to be true without

7. *Dred Scott v. Sanford* 60 U.S. (19 How.) 393 (1857).

8. 410 U.S. 113 (1973).

9. See, for instance, Ruth Bader Ginsburg, *Some Thoughts on Autonomy and Equality in Relation to Roe v. Wade*, 63 N.C. L. Rev. 375 (1985), Donald Regan, *Rewriting Roe v. Wade*, 77 Mich. L. Rev. 1569 (1979), Jed Rubenfeld, *The Right of Privacy*, 102 Harv. L. Rev. 737 (1989).

proof.”¹⁰ He then notes that a set of axioms “should be simple and consistent with one another,” as well as “logically independent.” (p. 24) With these definitions, then, it is incorrect to assert that an axiom is false within the logical system that it defines.

But Meyerson later forgets his own definition when he discusses *Dred Scott*. He notes that Taney’s decision rests on the axiom that African-Americans are inherently inferior to whites, and remarks that “One lesson from *Dred Scott* is that if you start with an incorrect axiom, you are unable to reason intelligently.” (p. 35) But this is a mischaracterization of what Taney did. He reasoned quite intelligently from the axioms that he started with. What Meyerson really means is that if Taney had chosen a different axiom, say the inherent equality of African-Americans and whites, then he would have come to a different conclusion.

While Meyerson has demonstrated that general logic is important in legal analysis, it leaves open the question of whether it should be considered uniquely mathematical. If so, then any discipline that proceeds in the western intellectual tradition can be said to benefit from mathematical thinking. That conclusion undercuts the novelty of Meyerson’s claims.

These quibbles aside, Meyerson’s beginning is a good one. He reminds us that it is important to make clear what one is assuming and what one is concluding in any argument, but particularly in legal ones, where it is easy to leave the hypotheses unstated. The more transparent the logic, the better it can be assessed.

TECHNICAL MATHEMATICS

It is in chapters 2-4 that Meyerson is best able to support his claim that mathematics can illuminate legal thinking. In these chapters he examines voting rules—including ruminations on the benefits of the electoral college—considers the super-majority aspects of the Constitution, and discusses the difficulties in apportioning seats in Congress. With interesting mathematics to be discussed and interesting law to ponder, this is the best part of the book. The only thing wrong is that there is not enough of it. Meyerson missed opportunities to put his material in a broader context and to include some additional mathematics, some even considered by the Supreme Court itself.

10. P. 24 citing John Daintith and R. David Nelson, *The Penguin Dictionary of Mathematics* 26 (Viking Penguin Press, 1989).

The theme of these chapters is voting and representation.¹¹ Mathematicians have been analyzing voting methods since the late 19th century, starting with the Marquis de Condorcet and his rival Jean Charles de Borda in pre-Revolutionary France. These efforts were directed toward trying to find fair methods of representation from an axiomatic standpoint. This intellectual thread led to Arrow's Impossibility Theorem in 1951. Meyerson does an excellent job explaining these ideas and showing their relevance to contemporary voting.

There have been a number of proposals to move to more sophisticated voting methods in order to address some of the problems that Meyerson discusses. Perhaps the most prominent proponent of alternative voting schemes is Lani Guinier,¹² who advocated cumulative voting (among other methods) as a way to ensure that the rights of minorities were not abused by the majority. In fact a number of local governments have implemented sophisticated voting schemes in order to ensure minority representation. In some instances these methods have been instituted as part of a settlement of suits brought under the Voting Rights Act.¹³ It would have been interesting to see how these methods relate to Meyerson's formal discussion of voting methods.

There is yet another thread of research in voting to which Meyerson only alludes: the question of measuring voting power. A voter is said to be pivotal in an election if her vote determines the outcome, that is, if a change in her vote would alter the outcome. For instance, in the last Presidential election, every state that voted for Bush was pivotal. The now-accepted measure of power in a voting system is the probability that any given voter will be pivotal. Meyerson mentions this briefly and refers to the resulting measure of power developed by Shapley and Shubik. (p. 52) While the Shapley-Shubik measure has some of the properties necessary to accurately measure voting power, the more

11. I will confine my remarks to the areas of mathematics discussed by Meyerson. It is important to be aware, however, that there are other areas of mathematics relevant to law as well. Probability theory has been used to assess the value of value of evidence for at least 35 years, notably in *People v. Collins*, 68 Cal. 2d 319 (1968) and has been the source of considerable legal commentary, most famously in Laurence Tribe, *Trial by Mathematics: Precision and Ritual in the Legal Process*, 84 Harv. L. Rev. 1329 (1970). More recently, the use of DNA evidence has been scrutinized in a similar way. See, for example, J. McKenna, J. Cecil and P. Coukos, *Reference Guide on Forensic DNA Evidence*, in *Reference Manual on Scientific Evidence* 273 (Federal Judicial Center, 1994).

12. Lani Guinier, *The Tyranny of the Majority* (Free Press, 1994).

13. For a general introduction and references see Samuel Issacharoff, Pamela S. Karlan, and Richard Pildes, *The Law of Democracy* (Foundation Press, 2001).

commonly accepted measure was developed by Banzhaf.¹⁴ Banzhaf developed this measure of power in 1964, in the wake of *Reynolds v. Sims*,¹⁵ as a way of deciding what voting schemes would meet the requirement of "one man-one vote." It is the Banzhaf measure that has been used in litigation, some of which has made it to the Supreme Court.

To see how Banzhaf's method works, consider the example that Meyerson uses to illustrate the use of a bicameral system. (p. 53) He considers a small republic with four states, Virginia, Massachusetts, Connecticut, and Rhode Island with populations 100, 80, 60 and 10 respectively. If representation is on a 10 to 1 basis then in the legislature they will have 10, 8, 6, and 1 representatives, respectively. It requires 13 votes to pass legislation. Finally suppose that state delegations vote as a bloc. What power does Rhode Island have in this voting scheme? The answer is 0. Why? There is no situation in which Rhode Island could cast a deciding vote. In order to get the requisite 13 votes to win, two of the three large states would have to cast votes in favor and that would ensure at least 14 votes, so no matter what Rhode Island does, the outcome would be the same.¹⁶

How many times will Virginia be pivotal? It will be pivotal in 4 instances, when it votes with Massachusetts (resulting in 18 votes in favor), with Connecticut (16), with Massachusetts and Rhode Island (19), and finally with Connecticut and Rhode Island (17). It will not be pivotal if it votes the same way as both Massachusetts and Connecticut, since in that instance, if Virginia were to change sides, the result would still pass since Massachusetts and Connecticut would together provide 14 votes to pass the legislation. A similar analysis would show that Massachusetts and Connecticut will be pivotal in 4 instances as well. This shows that all three of the larger states have the exact same amount of power (under the Banzhaf measure) even though they have differing numbers of votes. So we conclude that the power in this legislature is divided among the states with each large state getting 1/3 and Rhode Island getting 0.

14. For an explanation of why we should use the Banzhaf measure and not the Shapley-Shubik measure see Dan S. Felsenthal and Moshe Machover, *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes* 196 (Edward Elgar Publishing, 1998). There is some more really nice mathematics here, but it would be too much of a digression to deal with now.

15. 377 U.S. 533 (1964).

16. Meyerson says that Rhode Island "would rarely, if ever, be pivotal." (p. 53) In fact the answer is that it never has any chance.

But suppose we are interested in the power of the voters in the states, and not in the power of the states themselves. How should we compute the power of a citizen of Virginia? It turns out that the mathematics says that the power of an individual voter is the product of how often a particular voter is pivotal in electing a representative with the power of the state itself. It further happens that the likelihood of a voter being pivotal is proportional to the reciprocal of the square-root of the population. For example, in the case of Virginia, the power of an individual voter is proportional to $1/10$, the reciprocal of the square root of 100, and so the voter's total power in the legislature is $1/10 \times 1/3 = 1/30$.¹⁷

Banzhaf's methods first appear in litigation in *Whitcomb v. Chavis*,¹⁸ which is the most mathematically interesting opinion ever rendered by the Supreme Court. The issue before the Court was whether a multi-member district for the Indiana state legislature was constitutional. Banzhaf's method inspired a mathematical attack on the constitutionality of multi-member districts *per se*. The essence of the argument was described in the majority opinion this way:

In asserting discrimination against voters outside Marion County, plaintiffs recognize that Fortson, Burns, and Kilgarlin proceeded on the assumption that the dilution of voting power suffered by a voter who is placed in a district 10 times the population of another is cured by allocating 10 legislators to the large district instead of the one assigned to the smaller district. Plaintiffs challenge this assumption at both the voter and legislator level. They demonstrate mathematically that in theory voting power does not vary inversely with the size of the district and that to increase legislative seats in proportion to increased population gives undue voting power to the voter in the multimember district since he has more chances to determine election outcomes than does the voter in the single-member district.¹⁹

In an extended footnote²⁰ the majority outlined Banzhaf's theory, including a small example. Ultimately the majority rejected Banzhaf's arguments as being too "theoretical" and refused to find that multi-member districts were unconstitutional. Instead,

17. Felsenthal and Machover, *The Measurement of Voting Power* at 63 ff (cited in note 14).

18. 403 U.S. 124.

19. *Id.* at 144.

20. *Id.* at footnote 23.

they rested their analysis on a more traditional measure of representation by computing the number of people per representative and deciding that the spread in these numbers across districts exceeded the permissible variation.²¹

In his dissent, Justice Harlan reacted harshly to the innumeracy of his colleagues, commenting that "The only relevant difference between the elementary arithmetic on which the Court relies and the elementary probability theory on which Professor Banzhaf relies is that calculations in the latter field cannot be done on one's fingers."²² He proceeds to attack the validity of Banzhaf's model on a mathematical basis.²³ Particularly noteworthy in his critique is that his calculations at one point produce the number 120,000,000,000,000,000,000,²⁴ certainly the largest number ever to appear in a Supreme Court opinion.

This is not the only case in which Banzhaf's work appears. His methods were endorsed in *Ianucci v. Board of Supervisors*²⁵ as the way to measure power in a county board. The same methods were rejected in a challenge to the constitutionality of New York City's Board of Estimate²⁶ and in a challenge to the constitutionality of the Nassau County Board of Supervisors.²⁷ His work has also been used to show that the Electoral College is biased in favor of voters in large states even if it is biased in favor of the small states themselves.²⁸ It should be clear that there is a rich mathematical theory here with many important legal implications, which remains almost unknown in the broader legal community. Meyerson missed a perfect opportunity to remedy this.

The second of the technical mathematics chapters is devoted to the issue of super-majority rules in the Constitution.

21. Id. at 161-62.

22. Id. Justice Harlan, dissenting, at footnote 2.

23. Id. at 169.

24. Harlan claims this number is the ratio of the probability that a voter in Marion County, Indiana is pivotal if no voter is already committed to the candidate, to the probability that a voter is pivotal if 15,000 of the voters were committed to one candidate and 10,000 were committed to the other. Id. By my calculations the number is closer to 52,800,000,000,000,000,000.

25. *Ianucci v. Board of Supervisors of Washington County, NY*, 282 NYS 2d 502 (1967). While endorsing Banzhaf's method, the court misapplied the analysis. See Felsenthal and Machover, *The Measurement of Voting Power* at 99 (cited in note 14).

26. *Board of Estimate of City of New York v. Morris*, 489 U.S. 688 (1989).

27. *League of Women Voters of Nassau County v. Nassau County Board of Supervisors*, 737 F.2d 155 (1984).

28. See John F. Banzhaf, III, *One Man, 3.312 Votes: A Mathematical Analysis of the Electoral College*, 13 Vill. L. Rev. 304 (1968) and Guillermo Owen, *Game Theory* 212 (Academic Press, 2d ed. 1982).

The chapter contains an interesting historical discussion based on Madison's notes of the constitutional convention. In particular Meyerson notes that there was considerable discussion of whether the super-majority requirement should be set at $2/3$ or $3/4$ for overriding a veto. (p. 76) He also notes that the only time the requirement was set at $3/4$ instead of $2/3$ was in the instance of the number of states needed to ratify an amendment to the Constitution. (p.73) Unfortunately, Meyerson does not investigate further why the ratification supermajority is higher than the others. One possible explanation is the difference in baselines. The Articles of Confederation had required unanimous consent to make amendments, so the $3/4$ requirement was already a reduction. But the baseline for legislative acts was a simple majority, so requiring $2/3$ was already an increase.

He also neglects to mention that the bicameral structure of our government is inherently super-majoritarian. Buchanan and Tullock had already remarked on this in their seminal work in 1962²⁹ and it has been a theme throughout constitutional scholarship ever since.³⁰ It would have been very helpful if Meyerson had discussed some of Buchanan and Tullock's models in this chapter.

In the final technical chapter, Meyerson discusses the history and theory of the apportionment of Congress. The mathematics of apportionment is beautiful and Meyerson does a fine job of explaining it in such a confined space. It certainly would have benefited from a more leisurely exposition.³¹

These criticisms should not diminish the excellence of these chapters, however. Meyerson explains the mathematics clearly and shows how it applies to the Constitution. It is a unique contribution to the literature to have these subjects in one place and related to each other. I hope that this exposure of technical methods in legal analysis will lead to a wider appreciation for

29. James M. Buchanan and Gordon Tullock, *The Calculus of Consent*, 233-48 (U. of Michigan Press, 1962).

30. For a recent example see John O. McGinnis and Michael B. Rappaport, *Our Supermajoritarian Constitution*, 80 Tex. L. Rev. 703 (2002).

31. The bible of this subject is Michel L. Balinski and H. Peyton Young, *Fair Representation: Meeting the Ideal of One Man, One Vote* (Brookings Institution Press, 2001), which is a truly wonderful book devoted solely to the issues of apportionment and proportional representation. It contains a wealth of information, both historical and mathematical. For a briefer discussion of the history of the apportionment of Congress and the results under the 2000 census, see Paul H. Edelman and Suzanna Sherry, *Pick a Number, Any Number: State Representation in Congress after the 2000 Census*, 90 Cal. L. Rev. 211 (2002).

them and to an increased use of their methods where appropriate.

METAPHOR

We now come to the metaphorical uses of mathematics. Chapters 7-11 might be best described as ruminations on the Constitution inspired by mathematics. They employ ideas from chaos theory to incompleteness theory, from transfinite mathematics to topology. It is not productive to deal with each chapter separately as the shortcomings of this style of argument are pretty consistent. Instead I will focus on two of the chapters: Infinity and the Constitution (Chapter 8) and Constitutional Chaos (Chapter 10). Elsewhere I have commented on similar ruminations on the law using ideas from calculus.³²

Before dealing with specifics, I want to consider the different ways in which metaphor can be put to use as an explanatory device. One way is by relating an unfamiliar object to a familiar one. So when someone says "Alligator tastes just like chicken," this metaphor conveys some information if you haven't ever had alligator, but you have eaten chicken. Of course it would also be helpful if you've eaten alligator but not chicken. But this metaphor is more or less useless if you have eaten neither alligator nor chicken.

Imagine that you know nothing about either topology or federalism. How helpful was the earlier description of the relative power of the states in the federal constitution to you? If you know something about the law, then maybe you could get a hint of what topology is about. Or maybe a topologist might get some insight into federalism. The use of this metaphor is very much dependent on the audience for which it is intended.

A more sophisticated use of metaphor is available if someone knows both sides of the relationship. In such circumstances it may be possible to transfer information from one side to the other. For example, after Meyerson asks "Is the Constitution Chaotic" (p. 196) he goes on to find "... patterns that are reminiscent of those in modern chaos theory." (p. 197) Having established the analogy between chaotic systems and the Constitution, one can use more detailed information in the one as clues to what to look for in the other.

32. Paul H. Edelman, *A Tour of Mistakes*, 93 Nw. U. L. Rev. 343 (1998) (critiquing Steven Lubet, *A Tour of the Calculus of Justice*, 92 Nw. U. L. Rev. 1035 (1998)).

There are two difficulties with this more sophisticated use of metaphor. The first is that it requires the audience to be somewhat familiar with both sides of the metaphor. The other problem is that as a general rule the more one knows about one or the other side, the less persuasive these metaphors tend to be. It is, perhaps, an instance of a little knowledge being a dangerous thing.

Meyerson wants to employ metaphor in this more sophisticated way, but that presents a problem: the number of people who are knowledgeable in both law and mathematics is rather small. His solution is to assume his audience knows only about the Constitution and begin each of these chapters by explaining the relevant mathematics. The idea is to teach enough of the mathematics that the metaphors will be accessible and then discuss how the mathematical ideas inspire insights into constitutional analysis. The first part of this program is pretty successful. Meyerson's mathematical introductions are surprisingly good, especially given the limited space they are allotted. They are smoothly written and give a friendly introduction to lot of attractive mathematics. But then we get to the applications, and things start to run downhill.

Consider his chapter on infinity. (p. 148) Meyerson begins this chapter by summarizing the work of Cantor. Cantor starts from the premise that two sets of objects are said to have the same number of elements (the mathematical term is cardinality), if there is a one-to-one correspondence between the elements of one set and the other set. He shows that the set of counting numbers $\{1, 2, 3, \dots\}$ has the same cardinality as the set of even numbers by providing a one-to-one correspondence between them. He calls sets that have the same cardinality as the counting numbers countable. He goes on to mention the result of Cantor that the set of real numbers between 0 and 1 can not be put in one-to-one correspondence with the counting numbers and hence has a larger cardinality. This is all classic, pretty, mathematics that is not as widely known as it should be. He summarizes his discussion by relating the three relevant points about infinity:

The first is that you never can reach infinity; it goes on forever. Second, some infinite sets are the same size, even though they do not seem as if they should be. Finally, not all infinite sets are the same size; some are larger than others. (p. 152)

There is a problem with this synopsis. He uses "infinity" in the first sentence to mean something different than he did in his mathematical discussion. Meyerson defined infinity as the size of a certain kind of set. It doesn't make any sense to talk about it going on forever. It isn't going anywhere in his discussion. He can't even mean that the elements of the set are getting arbitrarily large, since in the case of the numbers between 0 and 1 they aren't. He is using "infinity" in this summary to mean something unbounded in some dimension. This makes his attempts to use the original definition of infinity somewhat suspect. Nevertheless, Meyerson then gives an application to legal analysis for each of the three points in his summary.

As an application of the assertion that "you never can reach infinity; it goes on forever," Meyerson discusses the fact that "the United States has a Constitution of infinite duration" (p. 152) and describes the debate among the Founders as to whether, in fact, the document was really of perpetual durability. What any of this has to do with the mathematical notions of infinity that Meyerson spent so much time on is left unclear. The section is closed with the remark that "Thus, in a somewhat Newtonian sense, the Constitution is infinite. Just as a body in motion will continue indefinitely unless some force acts on it, so will the Constitution stay in force until altered by 'some solemn and authoritative act.'"³³ Of course this "Newtonian sense" is again not the same notion of infinity as the one to which we were introduced.

Of course, all of the discussion about infinity obscures what would seem to be the more interesting constitutional question: is our current Constitution really the same as the one the Founders signed? If it isn't, then in what sense is the Constitution of "infinite duration?" Many have argued that the Constitution has been altered in ways that are not strictly constitutional.³⁴ It is particularly difficult to justify the claim that the pre-Civil War Constitution is the same as the Constitution including the Reconstruction Amendments.³⁵ So does the metaphor of "infinite duration" provide any useful information at all?

33. P. 155 quoting Federalist No. 78 (Hamilton).

34. See, for example, Sanford Levinson, *Accounting for Constitutional Change*, 8 Const. Comm. 409 (1991) and Bruce Ackerman, 1 *We the People: Foundations* (Harvard U. Press, 1991).

35. For a discussion of how the ratification of the 13th and 14th amendments violated Article V see Bruce Ackerman, *Constitutional Politics/Constitutional Law*, 99 Yale L.J. 453, 501 (1989). See also Mark E. Brandon, *Free in the World: American Slavery and Constitutional Failure* 201 (Princeton U. Press, 1998) ("Undoubtedly, though, taken to-

Let us look at Meyerson's third point, that "not all infinite sets are the same size; some are larger than others." He uses this observation to show that one thing of infinite value might be smaller than another.

For example, my life, to me, is of infinite value. I will do whatever I can to prolong my life, to be as healthy as I can be. . . . My children's lives, however, are worth far more to me than my own. (p. 160)

He then goes on to say that this provides a framework to discuss the abortion question, giving equal credence to the views of both sides.

Does infinity really play any role here? First it is important to note that while Meyerson claims that his life has infinite value to him, it is unlikely that he behaves in a way consistent with that belief. If he valued his life infinitely, he would do nothing which would put it at risk, since no matter how small the probability of harm, the expected value of such an action would be infinite. Thus, he wouldn't drive (cars are dangerous), he would probably hide in the basement (tornadoes, hurricanes, meteor showers), and eat only bread and purified water (pesticides and *E. coli*). I have no doubt that he values his life highly, but not infinitely.³⁶

But if he doesn't really value his own life at infinity, then there is no problem with him valuing the life of his children more (although also finitely, since I presume that he drives them in a car on occasion). So we are back to the mundane comparison of large finite numbers. Once again, the metaphor is not very helpful.

Meyerson illustrates his second point, that "some infinite sets are the same size, even though they do not seem as if they should be," by the notion of "infinite rights." (p. 155) "The concept of infinity helps explain why some constitutional conflicts are so difficult to resolve. When dealing with finite quantities, we can decide readily which is greater. But in the realm of fundamental rights and liberties, we are dealing with freedoms of infinite value for which simple comparisons may be impossible." (pp. 155-56)

gether, the Amendments' principles were the core of a fundamentally different political world.")

36. Just a page before he discusses the infinite value of life, Meyerson relates a similar argument used by Learned Hand to decide when the danger to society is sufficient to justify a burden on free speech. (p. 159)

As already discussed it is unlikely that freedoms are really of "infinite value," whatever that might mean.³⁷ But even if these freedoms were of infinite value, it is not clear what the implications are. Meyerson believes that it works in support of the protection of rights:

Recognizing the infinite value of speech is also helpful in explaining the need to be vigilant against even the smallest restrictions on free expression. A fraction of infinity still equals infinity. The loss of even a fraction of the right of free expression imposes a burden of infinite scope. (p. 157)

But that is certainly not the only way to view the matter. If half of infinity is still infinity then a fractional burden on the right of free expression leaves one with exactly the same amount of freedom, doesn't it? If the value of this reduced right is the same as the original then no harm is done. Is this any less persuasive than Meyerson's own argument?

Moreover, none of this is really useful in analyzing government burdens on individual rights. Meyerson concludes that "In a complicated world, we must recognize that there frequently will be situations where more than one interest of infinite value is at stake. Simplistic comparisons must, therefore, of necessity give way to a far more sensitive evaluation of these competing interests." (p. 159) It is difficult to argue with this sentiment, but what is gained by talking about infinity? Is the usual discourse of balancing of rights and undue burden insufficient? Meyerson never explains further.

Ultimately, at the end of this chapter, the reader has been introduced to some pretty mathematics, but the subsequent legal musings are inconsistent with the presentation of the mathematics, misapply the mathematics at times, and ultimately fail to demonstrate any usefulness for the metaphor proffered. If anything, the mathematical concepts have led to muddier thinking than before.

As a second example of Meyerson's use of mathematical metaphor, let us look at his discussion of chaos theory and the Constitution. Chaos theory is a generic name for a number of related mathematical topics. Meyerson attempts to give an introduction to the topic in twelve pages. I am not sure how helpful it

37. In this section he still is not using the term "infinity" as the size of a set, but rather as a shorthand for unbounded.

really is to someone who has little previous knowledge.³⁸ Terms get introduced with little or no definition (strange attractors and bifurcation) and topics having nothing to do with chaos are mentioned (catastrophe theory). Nevertheless, since no real mathematics is employed in the legal discussion, the technical description is irrelevant.

Ultimately Meyerson characterizes chaotic systems as deterministic systems defined by simple rules that lead to complex behaviors. In particular, these systems are so sensitive to initial conditions that "long-range prediction is impossible." (p. 192) Meyerson then poses the question "How meaningful is it to say that the Constitution is chaotic?" and after some caveats concludes "the entire issue of constitutional interpretation by the Supreme Court has much in common with a dynamic, chaotic system." (p. 197) He goes on to tell two different and somewhat contradictory stories about constitutional interpretation.

The first story is a "chaotic" explanation of the intricacy of current constitutional doctrine. "Under the doctrine of *stare decisis*, each decision builds on previous ones. . . . [E]ach ruling depends on how the Court ruled the previous time a similar case was decided. Moreover, . . . the longer the time span under consideration, the more cycles of iteration occur and the greater the likelihood of complexity." He illustrates it thus: ". . . [T]he Court's striking down of a law barring parents from sending their children to private school led to its declaration that bans on the sale of contraceptive devices to married couples were unconstitutional, which, in turn, led to *Roe v. Wade*." He summarizes this discussion with the quote "'How can simple rules lead to complex phenomena? Via long runtime.'" ³⁹

There are at least two objections one might make to this story. The first is that Meyerson assumes a very mechanistic form of legal rulemaking in which the outcome of each case is completely decided by previous decisions. He has to take this view if he wants to view constitutional interpretation through a chaotic lens, because his chaotic model is dependent on the system being deterministic. But this view has come under intense attack and has largely been discredited.⁴⁰

38. There is a particularly nasty typo in the table on page 190 in which, I presume, the parameter a is replaced by λ .

39. P. 198 quoting Rudy Rucker, *Mind Tools* 314 (Houghton Mifflin Company, 1987).

40. See, generally, Daniel A. Farber and Suzanna Sherry, *Desperately Seeking Certainty: The Misguided Quest for Constitutional Foundations* (U. of Chicago Press, 2002).

The second objection one might raise is to the assertion that the outcomes of constitutional interpretation have become increasingly complex. What does such an assertion mean? Certainly the issues before the Court are more intricate but does that mean the system itself is more complex? Without some sort of definition of complexity it is hard to know. Meyerson is not helpful in this regard.

The second story that Meyerson tells is considerably different. He asserts that "the fact that the current state of constitutional doctrine is vastly different from what the framers would have envisioned should not be considered surprising" in the light of chaos theory. (p. 199) He explains this by saying that the lack of precise understanding of what the framers would have wanted results "in a very different constitutional path from what the framers ever would have expected." (p. 199) He concludes that "[i]t is unrealistic to expect that our current doctrine would fulfill the framers' expectations. Even were society not to change, the long-term iteration of inevitably imperfect decisions would tend to lead to results that were unforeseeable initially." (p. 200)

The first question we might ask is whether it is true that the current state of constitutional interpretation is all that different from what the framers would have envisioned if they were privy to all that went before. Meyerson never gives any support for this position, but it certainly warrants some sort of justification. Even if it is significantly different, is the discrepancy getting larger or smaller over time? Again Meyerson is silent.

And even if we were to accept the truth of these assertions, it is not clear that the constitutional regime Meyerson describes is consistent with chaos theory as he describes it. If the Constitution is chaotic, then it is deterministic, *i.e.*, once it gets going everything is decided. We saw this in the first story that Meyerson tells. There is no room in this description for current judges to get the answer to a legal question wrong. If the current state of constitutional interpretation is not what the framers would have thought, it must be because they had an imprecise knowledge of the "initial conditions" of the Constitution, not because current judges misinterpreted their intent.

The rest of this chapter is very similar to the earlier parts. Meyerson invokes more ideas from chaos theory (fractals, strange attractors, stability) as springboards for analyzing the Constitution. The inspirations are not always consistent with each other, and the legal positions are at best undeveloped. In-

evitably the reader remains unconvinced that chaos theory really sheds any light whatsoever.

These two chapters are, I think, representative of the Meyerson's use of mathematical metaphor. While the introductory mathematical material can be interesting, the application to the law is too often strained or just inapt. Mostly they seem to serve as cover for undeveloped legal musings. Ultimately one is left unsatisfied.

CONCLUSION

Can mathematics be used to inform legal analysis? The answer, like that of many such questions, is yes and no. Certainly the logical rigor of mathematical thought is helpful in analyzing legal arguments in the same way that it is useful to analyze any other sort of argument. True technical mathematics can be useful, indeed almost necessary, in analyzing a relatively small, but interesting, class of legal problems. But mathematical metaphor seems to be of little use and perhaps only serves to confuse.⁴¹

It would be churlish, however, to dismiss *Political Numeracy* this quickly, for it has more than just legal analysis to offer the reader. Meyerson clearly enjoys mathematics and does a fine job conveying that enthusiasm. His exposition of mathematical topics is very accessible and is, if not completely accurate, faithful to the subject. It provides an entry to some very beautiful mathematical topics with which every educated person should be familiar (but most are not). In this innumerate world, that is a substantial accomplishment and worthy of respect in itself.

41. It is worth considering whether mathematical metaphor is inherently less useful than, say, sports metaphors. I would say that it is likely to be less useful for two reasons. First, the likelihood of the mathematics being misapplied is probably higher, and second, the patina of certitude that mathematics provides discourages a critical view of the metaphor. But, cf. Mark A. Graber, *Law and Sports Officiating: A Misunderstood and Justly Neglected Relationship* 16 Const. Comm. 293 (1999).